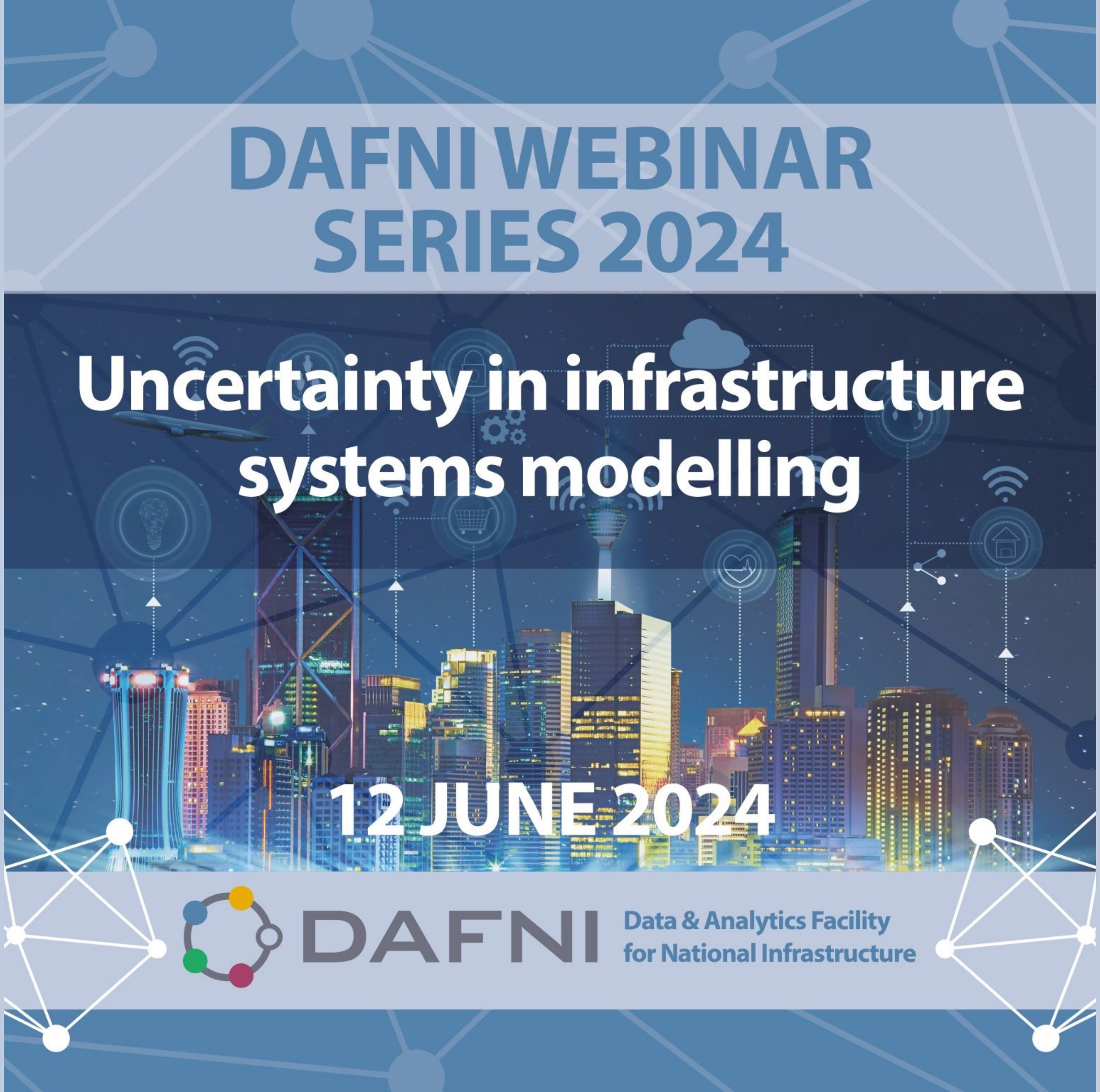
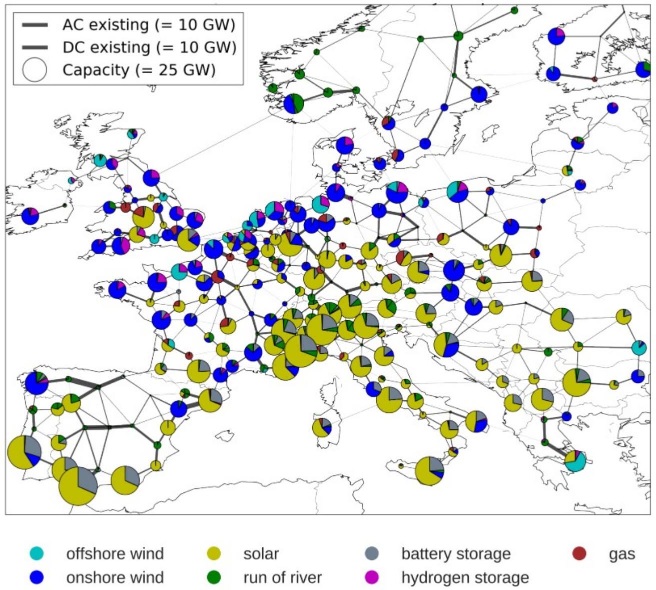


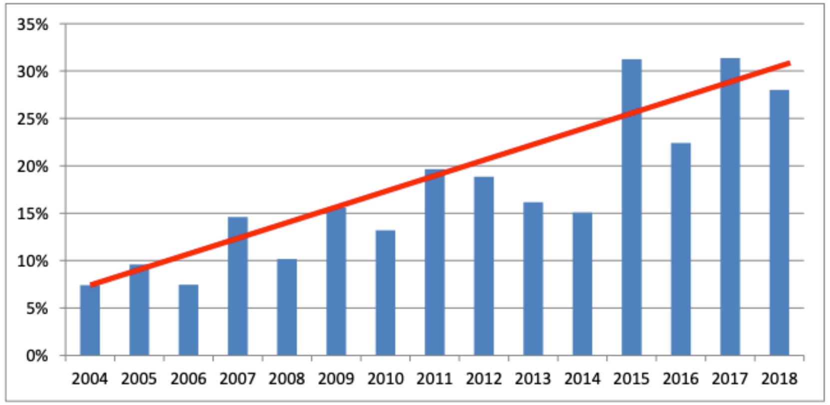
Francesca Pianosi
Saskia Salwey
University of Bristol

Mathematical models are increasingly used to inform decisions in a variety of sectors



Brown et al 2018 doi: 10.5334/jors.188

Share of EC Impact Assessments supported by modelling



[JRC 2019 *Modelling for EU Policy support*]

However, model outputs are conditional on many uncertain assumptions

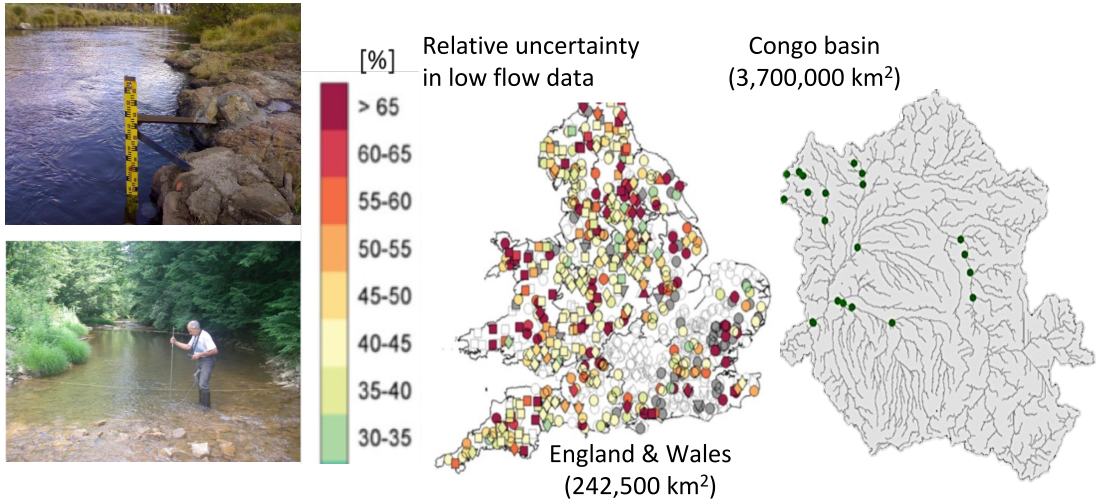
-- about the system’s properties and drivers *now*

due to errors and gaps in data used to build and run models, and simplifying assumptions whose adequacy is difficult to establish

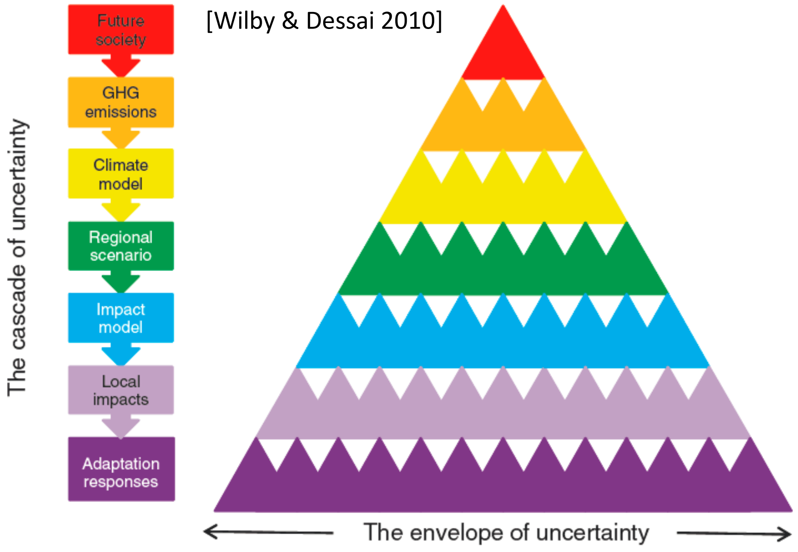
-- about how drivers will evolve in the future

“it’s difficult to make predictions, particularly when they concern the future” (Danish proverb)

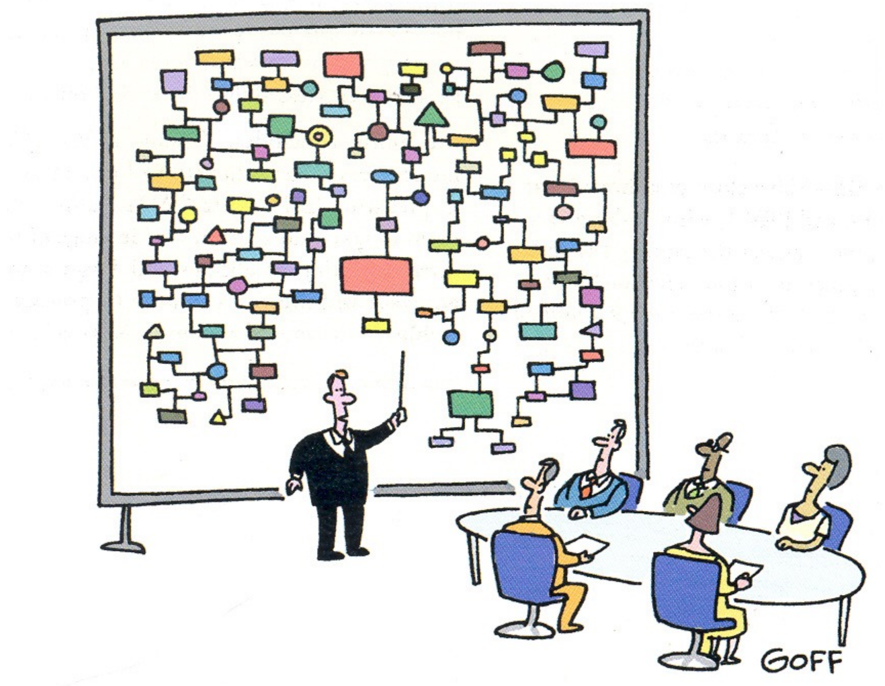
Example: river flow data



[Wilby & Dessai 2010]



For models to be trustworthy and effective we must



-- avoid **spurious precision**

-- identify **key sources of uncertainty**

when does the model stops being valid?

where to start in order to improve the model?

-- identify **robust designs**

which designs perform "well enough" across a range of future scenarios?

Uncertainty & sensitivity analysis provide a generic methodology to explore these questions

-- Uncertainty analysis (or Uncertainty Quantification):

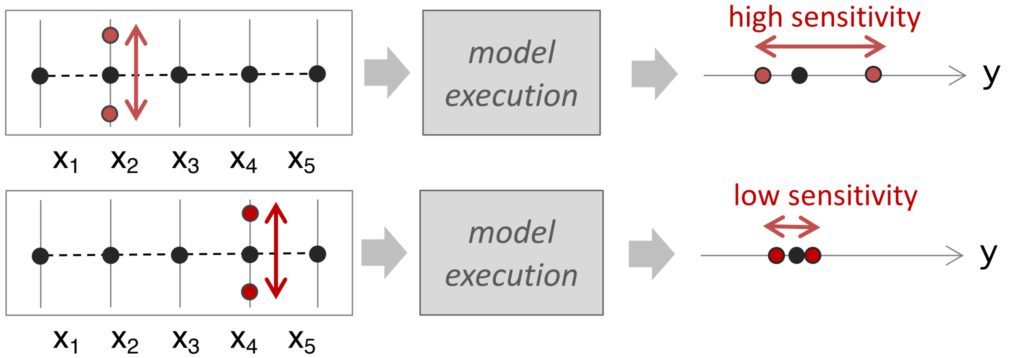
What is the range of variability of the model outputs given our level of uncertainty in the model input data and assumptions?

-- Sensitivity analysis (or Uncertainty Attribution):

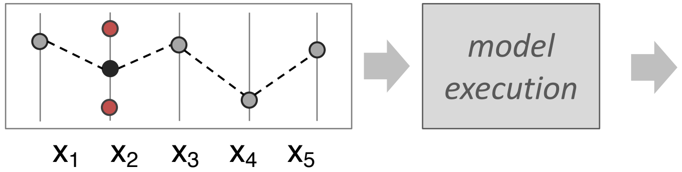
Which uncertain input mostly contribute to the variability of model outputs, when and where?

In this seminar, we focus on “Global” approaches to uncertainty and sensitivity analysis

Local Sensitivity Analysis:
investigates the effects of varying uncertain inputs **one at the time** around a **baseline** point

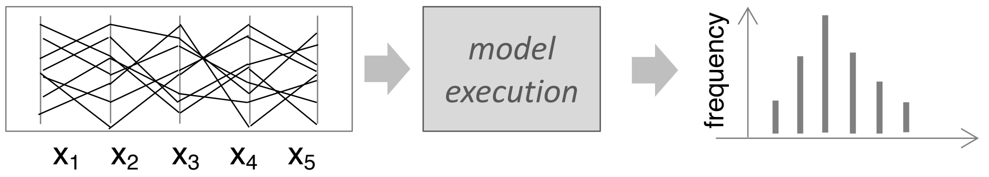


However…



How different would sensitivity assessment be if using a **different baseline**?

Global Sensitivity Analysis:
investigate the effects of varying all uncertain inputs **simultaneously** across their **entire variability space**



Where is this uncertainty mostly coming from?

What is the relative contribution of x1, …, x5 to the uncertainty in y?

Global approaches are based on repeated executions of the model against different inputs' combinations and a statistical analysis of the resulting input-output dataset

[1] characterize uncertainty in input factors

[2] propagate input uncertainties through the model

[3] quantify output uncertainty

[4] attribute output uncertainty

RUNNING
MONTE-CARLO SIMULATIONS

ANALYSING
INPUT-OUTPUT DATASET

characterize uncertainty in the inputs

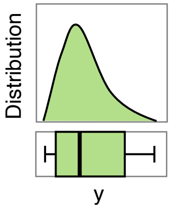
(e.g. via a list of possible values, variability range, probability distribution.)

sample N combinations of inputs

(e.g. via random sampling, Latin Hypercube sampling)

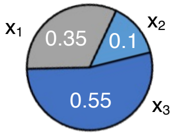
execute the model against each inputs' combination

derive the output(s)' distribution or CIs



calculate sensitivity indices

contribution to output uncertainty



- Elementary Effects method
- Variance-based (Sobol') method
- etc.

WHY

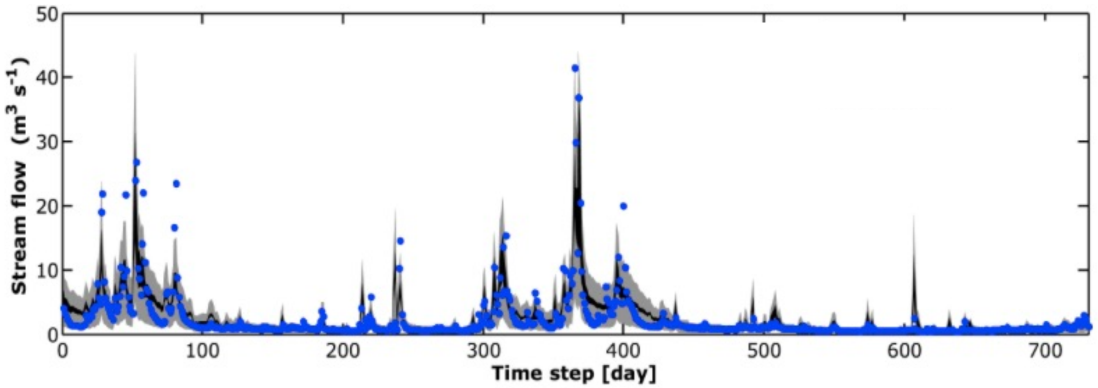
doing UA/SA?

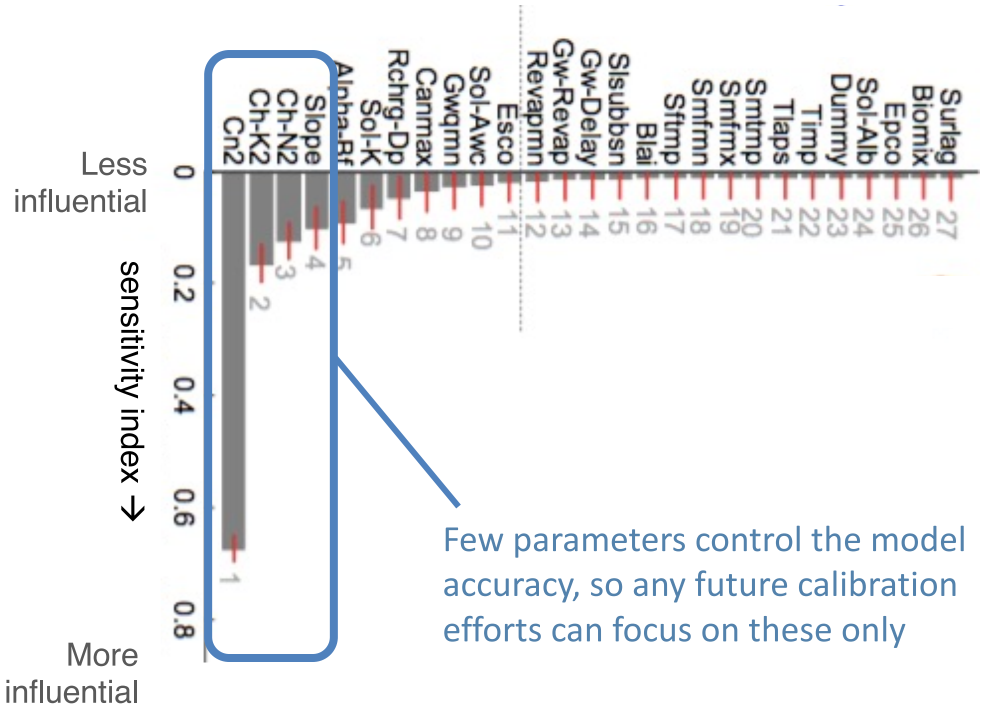
Guiding the calibration of the model

Which parameters mostly control the model's prediction accuracy and therefore should be the focus of computationally-expensive calibration?

Application to SWAT hydrological model

Zadeh et al 2017
doi: 10.1016/j.envsoft.2017.02.001



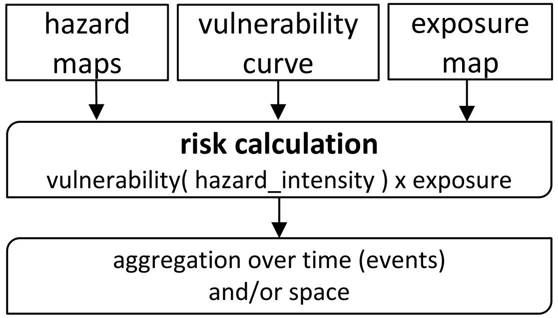


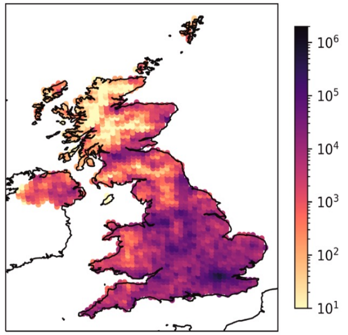
Prioritizing efforts for uncertainty reduction

Application to risk assessment model

Dawkins et al 2023
doi: 10.1016/j.crm.2023.100511

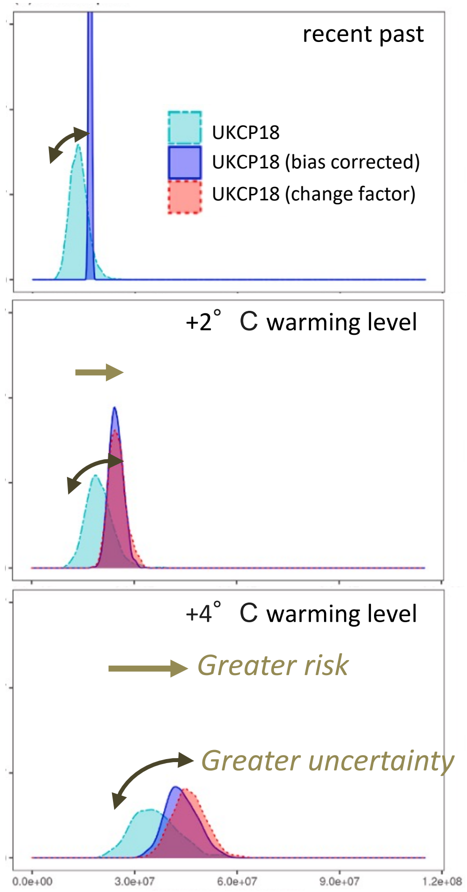
Which source of uncertainty influence the precision of model output the most?
Where efforts for model/data improvement will have most significant effects towards improving precision?
(... and where they won't?)





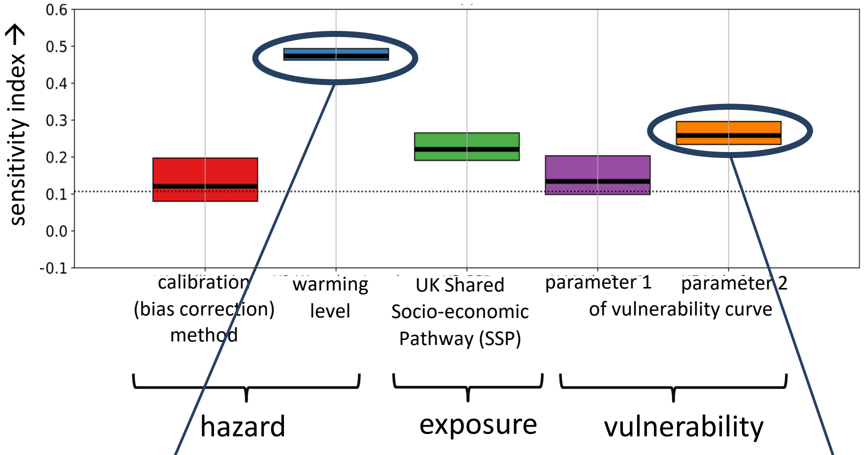
Risk: Expected Annual Impact (EAI): number of days of work lost due to heat stress

Quantifying future risk
(using different hazard maps)



Risk: spatially aggregated EAI

Quantifying sensitivity of risk (mean of spatially aggregated EAI) to input uncertainties



warming level most influential factor

vulnerability curve parameter 2nd most influential (more than bias correction!)

Model evaluation

Wagener et al 2022
On the evaluation of climate change impact models *WIREs-CC*
doi:10.1002/wcc.772

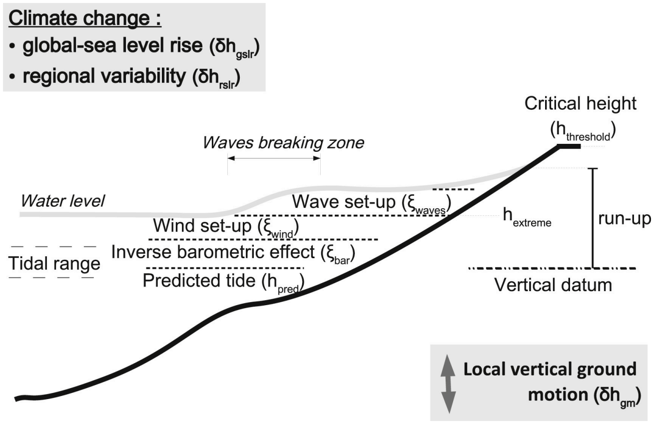
What are the dominant control of the model output?
Are model outputs sensitive to decision-relevant inputs?

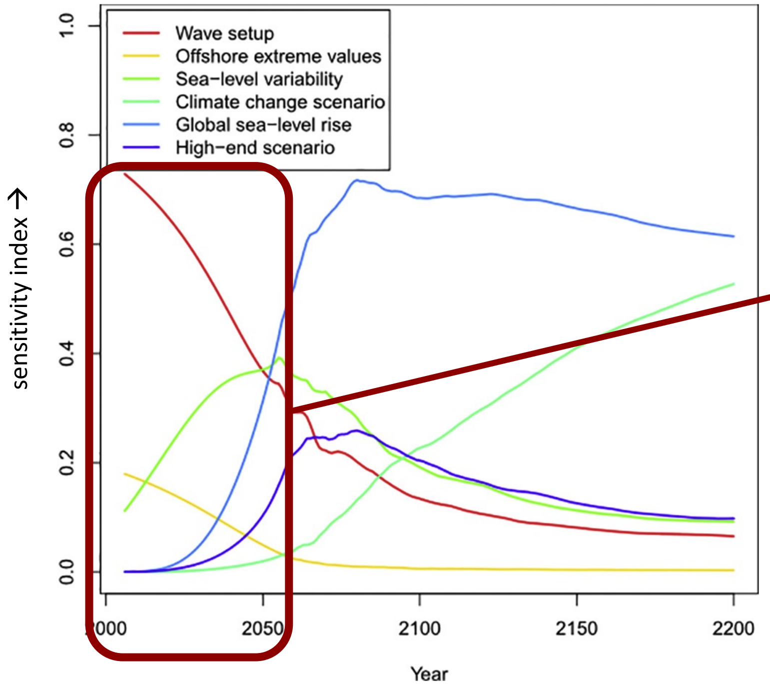
If outputs are more strongly controlled by uncertain assumptions/parameters than by the policy/scenario inputs, then the model will tell us more about the consequences of the assumptions embedded in it than it will tell us about the different policy options/scenarios

Application to flood defence assessment model

Le Cozannet et al 2015
doi: 10.1016/j.envsoft.2015.07.021

Sensitivity of coastal defence vulnerability
(= annual probability of exceeding the threshold height of coastal defences)





In the 'mid-term', the model predictions are controlled by the modeller's choice of the (very uncertain) 'wave setup' parameter way more than they are by climate change and sea level rise scenarios

→ the model should not be used for impact assessment on such temporal scales

Example

UA&SA on DAFNI: a hydrological model example

Uncertainty Quantification and Sensitivity Analysis for Resilient Infrastructure Systems (USARIS)

USARIS will set the foundations to include UA&SA into DAFNI and demonstrate their value to the DAFNI users' community

- We will rely on existing UA&SA packages (e.g. https://safetoolbox.github.io/)
- The integration into DAFNI will be conducted by developing **two pilot applications** (DAFNI workflows) in water and energy
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Ultimately USARIS will contribute to enable and promote best practices for responsible modelling in the DAFNI users community

Pianosi
UQ&SA
water systems

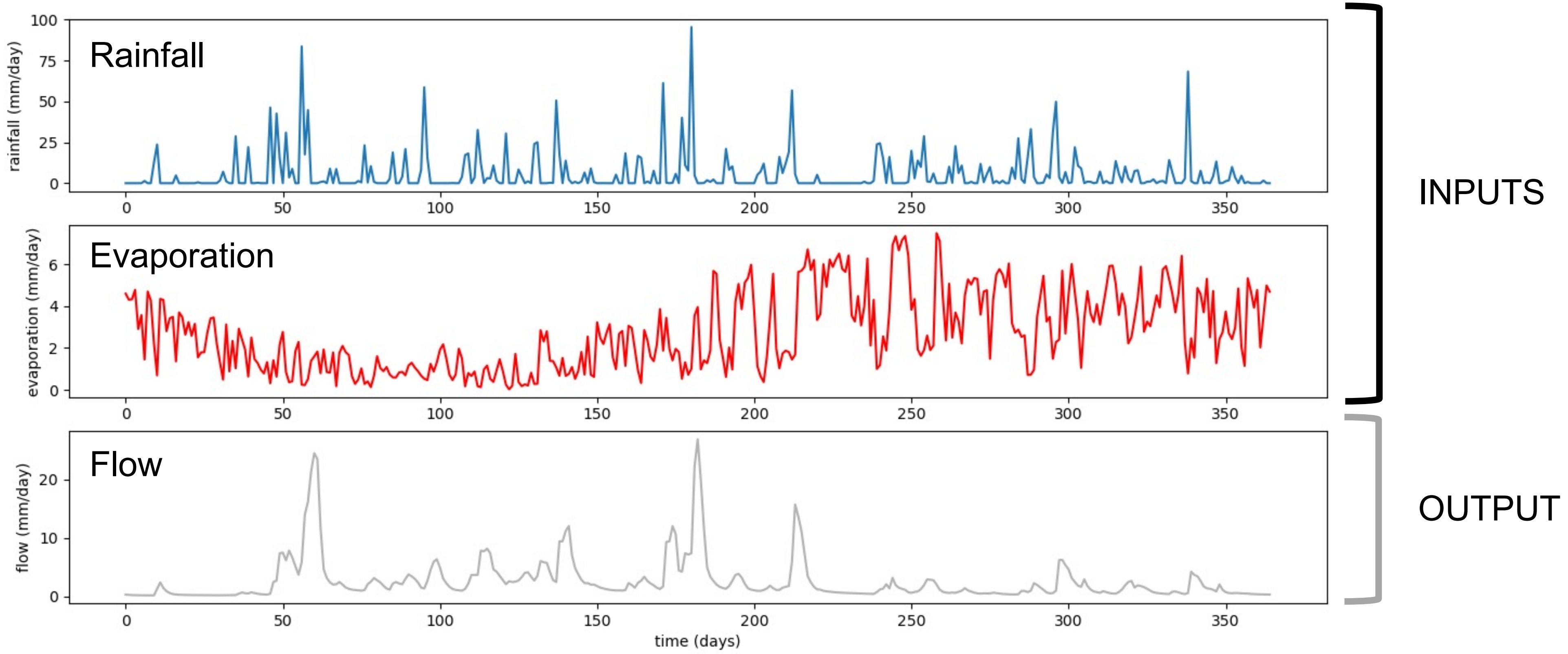
Salwey
UQ&SA
water systems

Bloomfield
energy systems

Coxon
water systems

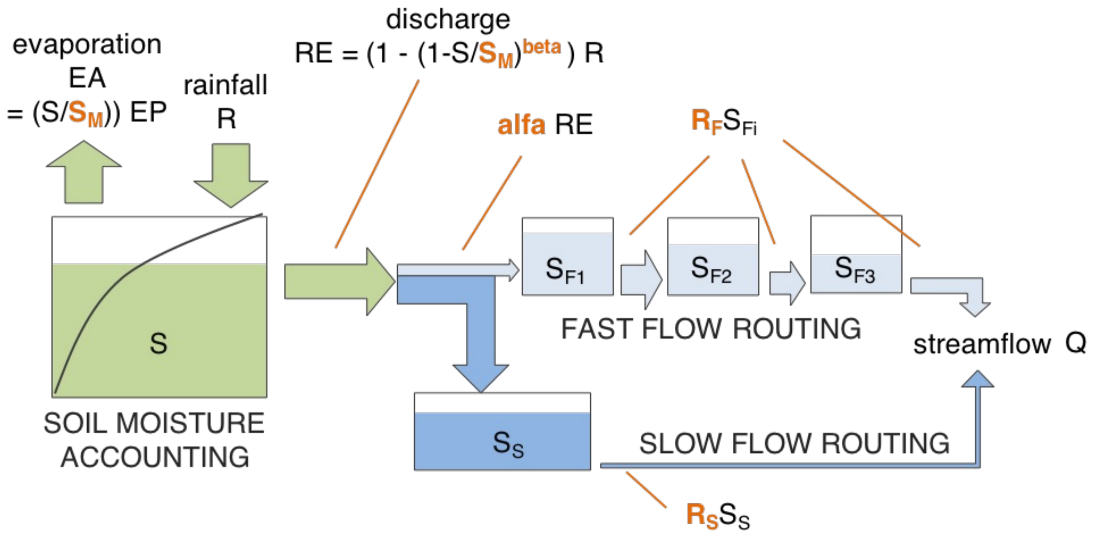
Hydrological Model: HyMod

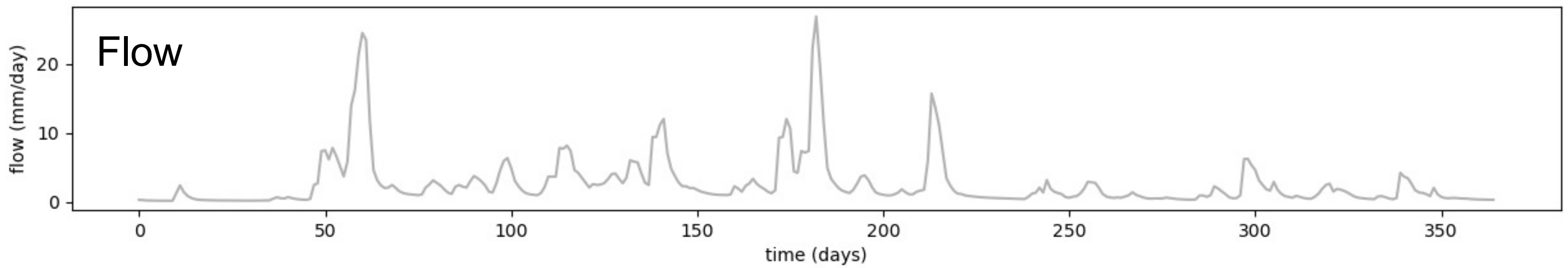
A hydrological model takes rainfall and evaporation over a river basin and returns timeseries of river flow.



Hydrological Model: HyMod

This simple model has 5 parameters which control the transformation of rainfall into river flow.

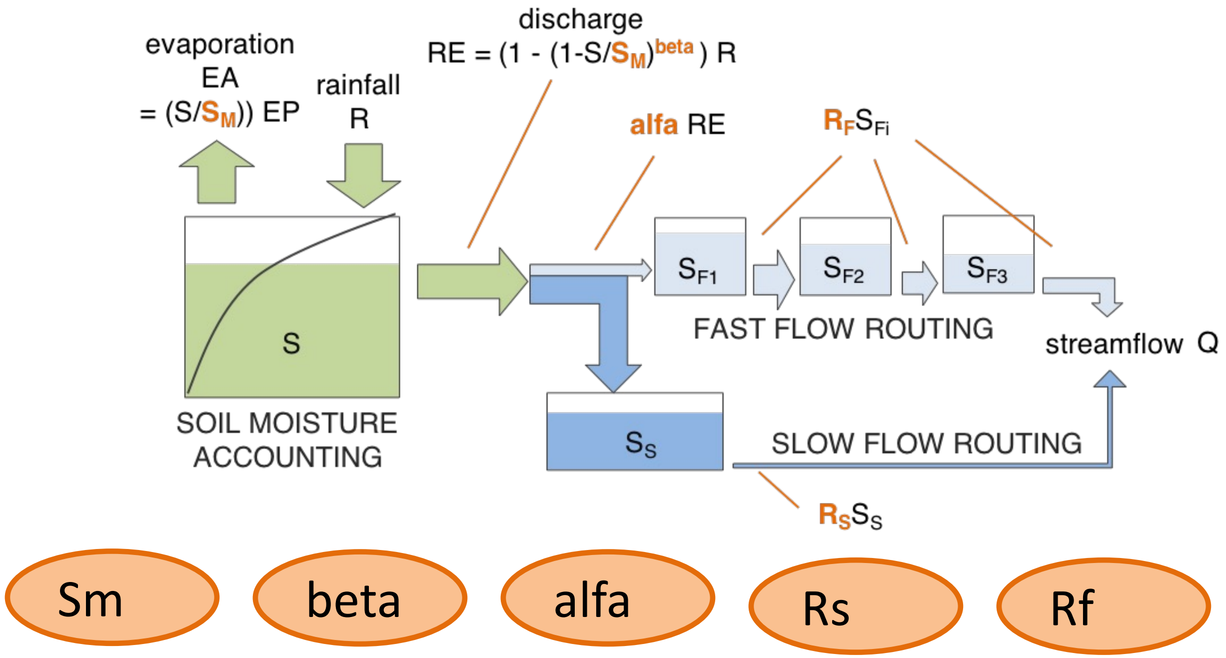




OUTPUT

Hydrological Model: HyMod

This simple model has 5 parameters which control the transformation of rainfall into river flow.



We can use UA & SA to investigate how uncertainty in the input parameters translates to variability in model outputs

Model application: (1) flood defences (2) water supply system

Running Montecarlo simulations on DAFNI

(1) Upload model and data to the DAFNI platform
(2) Write DAFNI workflow to run the model
(3) Sample parameter space by **looping** over workflow
(4) Visualise outputs

Running Montecarlo simulations on DAFNI

(1) Upload model and data to the DAFNI platform
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(4) Visualise outputs

Home / Models / SAFE-Hymod-test

SAFE-Hymod-test Latest

Versions: View all | No default | Go to latest

Input parameters

Parameters available to control each run of this model.

Title	Name ↑	Type	Min	Max	Default	Required?
alfa	alfa	number	0	2	0.7	Yes
beta	beta	number	0	2	0.5	Yes
Rf	Rf	number	0.1	1	0.6	Yes
Rs	Rs	number	0	1	0.05	Yes
Sm	Sm	number	0	400	200	Yes

Rows per page: 5 1-5 of 5

Running Montecarlo simulations on DAFNI

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(2) Write DAFNI workflow to run the model
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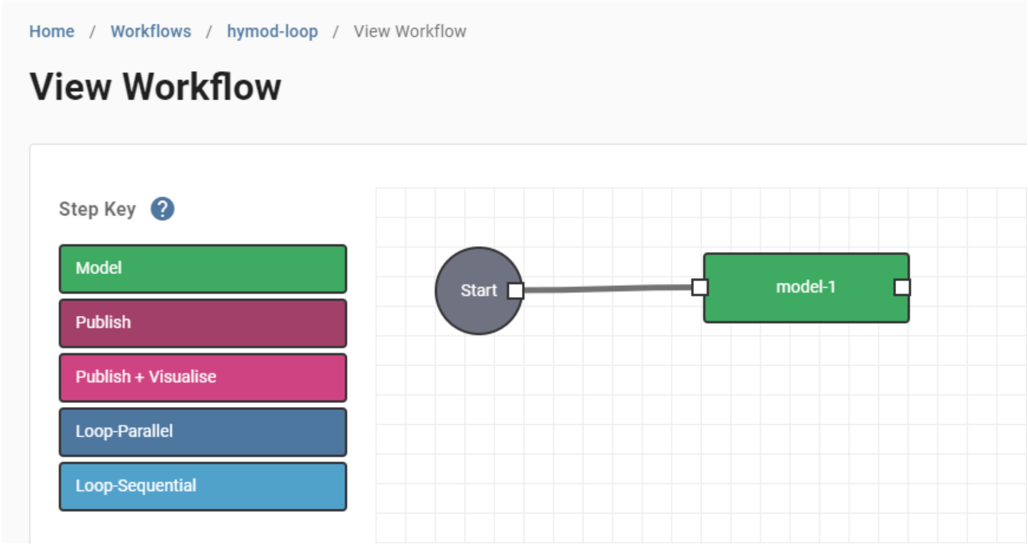

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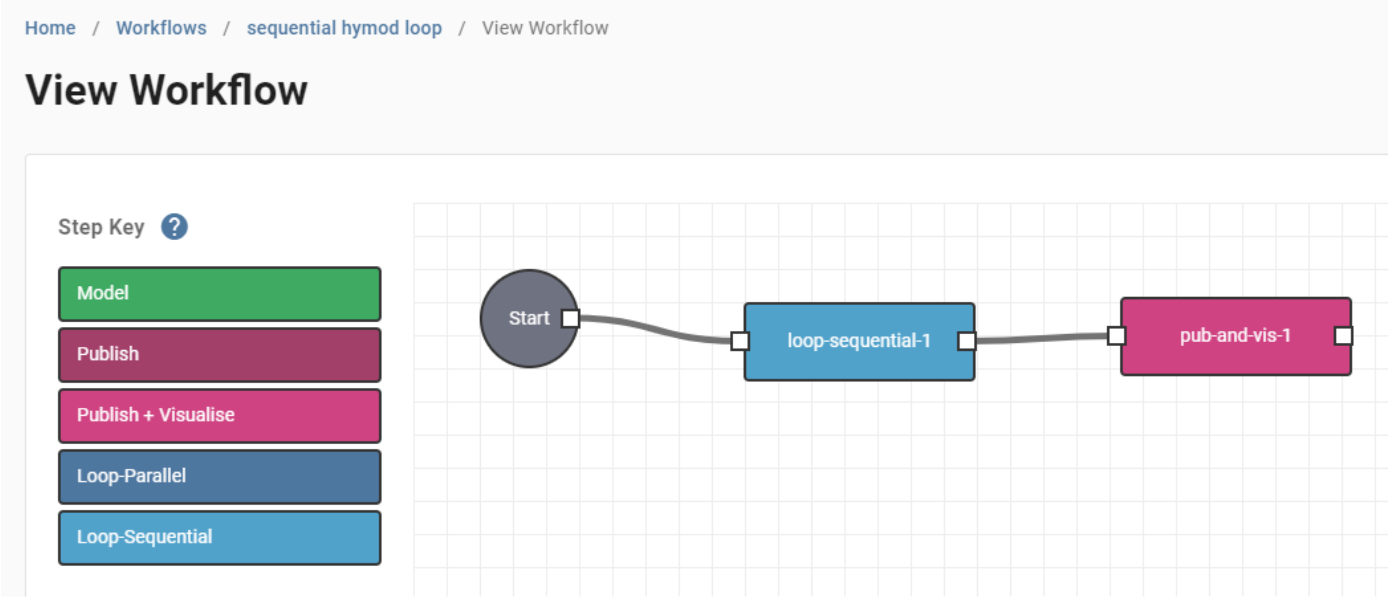

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(1) Upload model and data to the DAFNI platform
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(3) Sample parameter space by **looping** over workflow
(4) Visualise outputs

Data files

	File name	File size	Format
☐	loop-all-model-1-0-flow_sim.csv	9.13 kB	CSV
☐	loop-all-model-1-0-params.csv	125 B	CSV
☐	loop-all-model-1-1-flow_sim.csv	9.13 kB	CSV
☐	loop-all-model-1-1-params.csv	125 B	CSV
☐	loop-all-model-1-10-flow_sim.csv	9.13 kB	CSV

Download selected files   Rows per page: 5   1-5 of 54

Parameters to iterate

Choose the parameter to loop over and the steps that contain it:
Parameter name *: Sm
Select steps *: model-1
☑ Generate values
Count *: 50
Distribution *: Uniform
Minimum *: 0
Maximum *: 400
Delete parameter

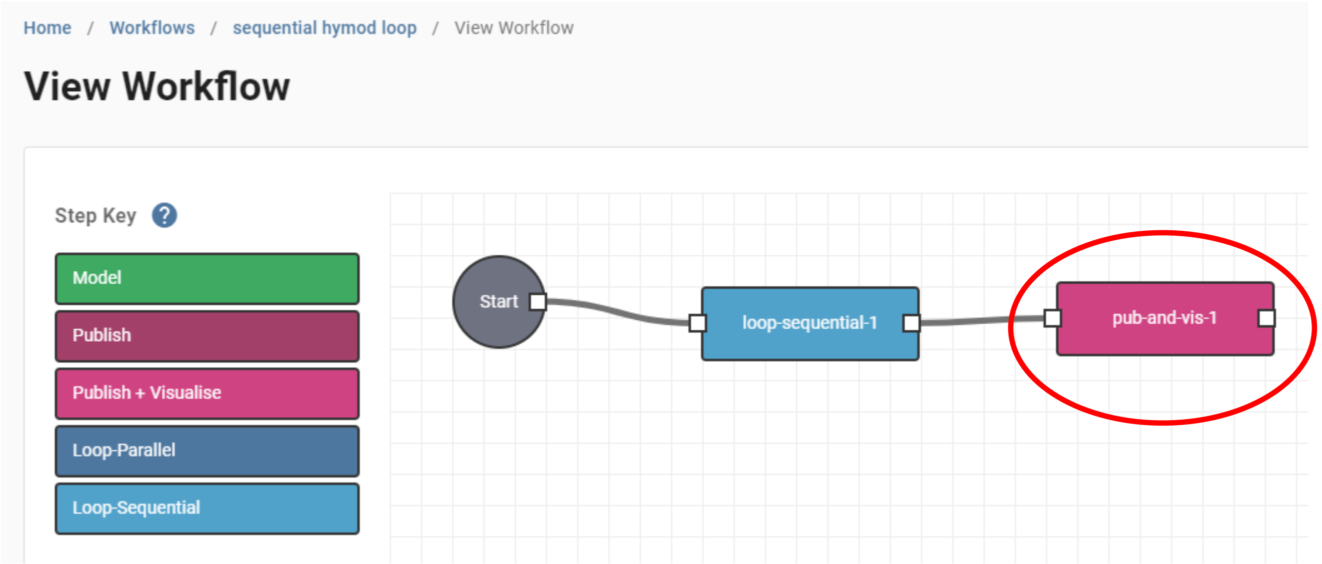
Choose the parameter to loop over and the steps that contain it:
Parameter name *: beta
Select steps *: model-1
☑ Generate values
Count *: 50
Distribution *: Uniform
Minimum *: 0
Maximum *: 2
Delete parameter

Running Montecarlo simulations on DAFNI

(1) Upload model and data to the DAFNI platform
(2) Write DAFNI workflow to run the model
(3) Sample parameter space by **looping** over workflow
(4) Visualise outputs

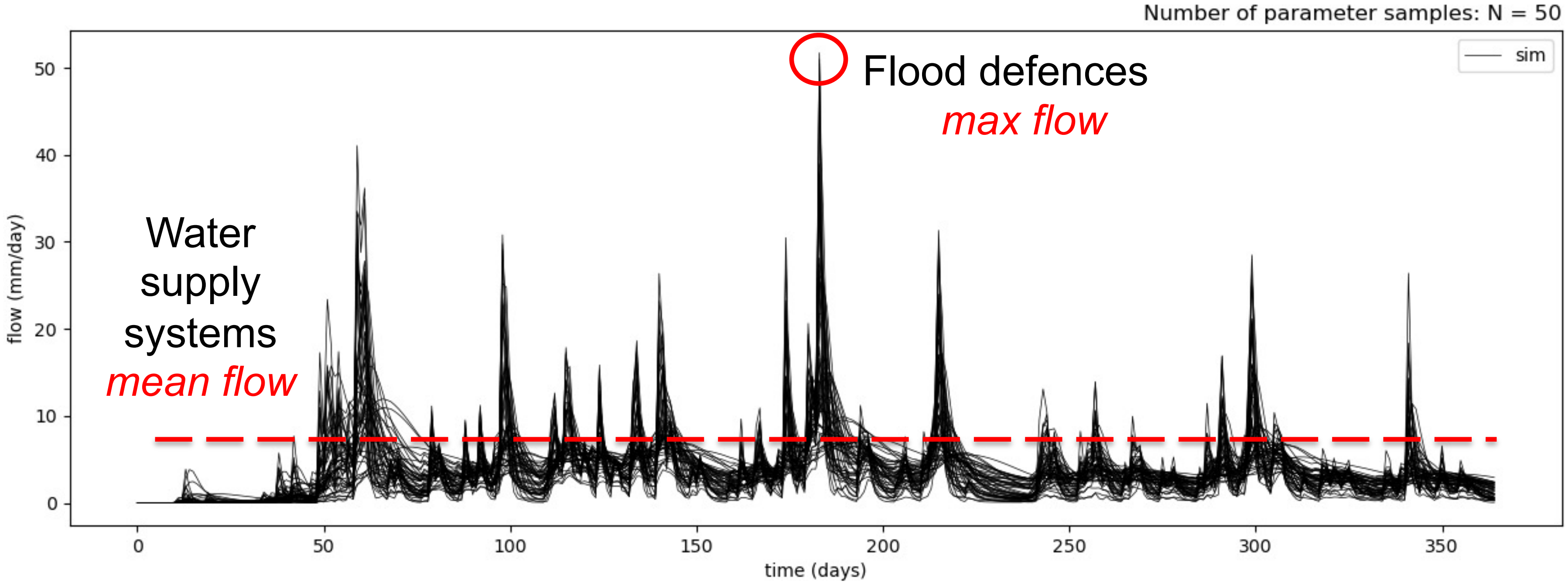
jupyter

Model outputs can be accessed and analysed in a jupyter notebook



Analyzing input-output dataset

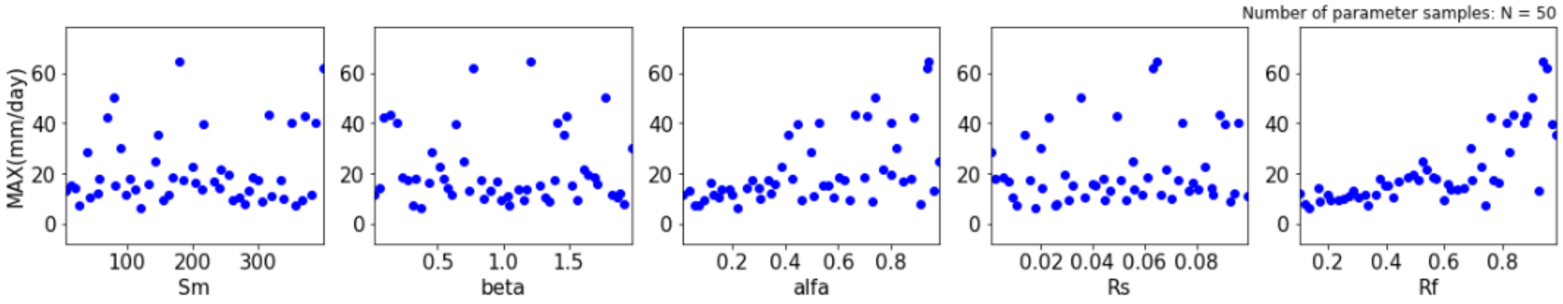
```
# Plot Monte Carlo (MC) simulations results:
plt.figure(figsize=[15,5])
plt.plot(np.transpose(QQ), 'k', linewidth = 0.5)
plt.ylabel('flow (mm/day)'); plt.xlabel('time (days)')
plt.legend(['sim'])
plt.title("Number of parameter samples: N = %d" % N, loc='right')
plt.show()
```



Analyzing input-output dataset

In [23]:
```
# Calculate MAX and MEAN for each parameter sample:
YY = np.nan * np.ones((N, 2))
YY[:, 0] = QQ[:, warmup:365].max(axis = 1)
YY[:, 1] = QQ[:, warmup:365].mean(axis = 1)

# Define interactive visualisation function to produce scatter plots for the chosen output metrics:
def scatter_function(metric='MAX'):
    if metric == 'MAX':
        i = 0
    elif metric == 'MEAN':
        i = 1

    # Extract output metric:
    Y = YY[:, i];
    Y_Label = metric + '(mm/day)'

    # Scatter plots of the output metric against input samples:
    plt.figure(figsize=[20,3])
    pf.scatter_plots(X, Y, Y_Label=Y_Label, X_Labels=X_Labels)
    plt.title("Number of parameter samples: N = %d" % N, loc='right')
    plt.show()

interact(scatter_function, metric = ['MAX','MEAN']);
```

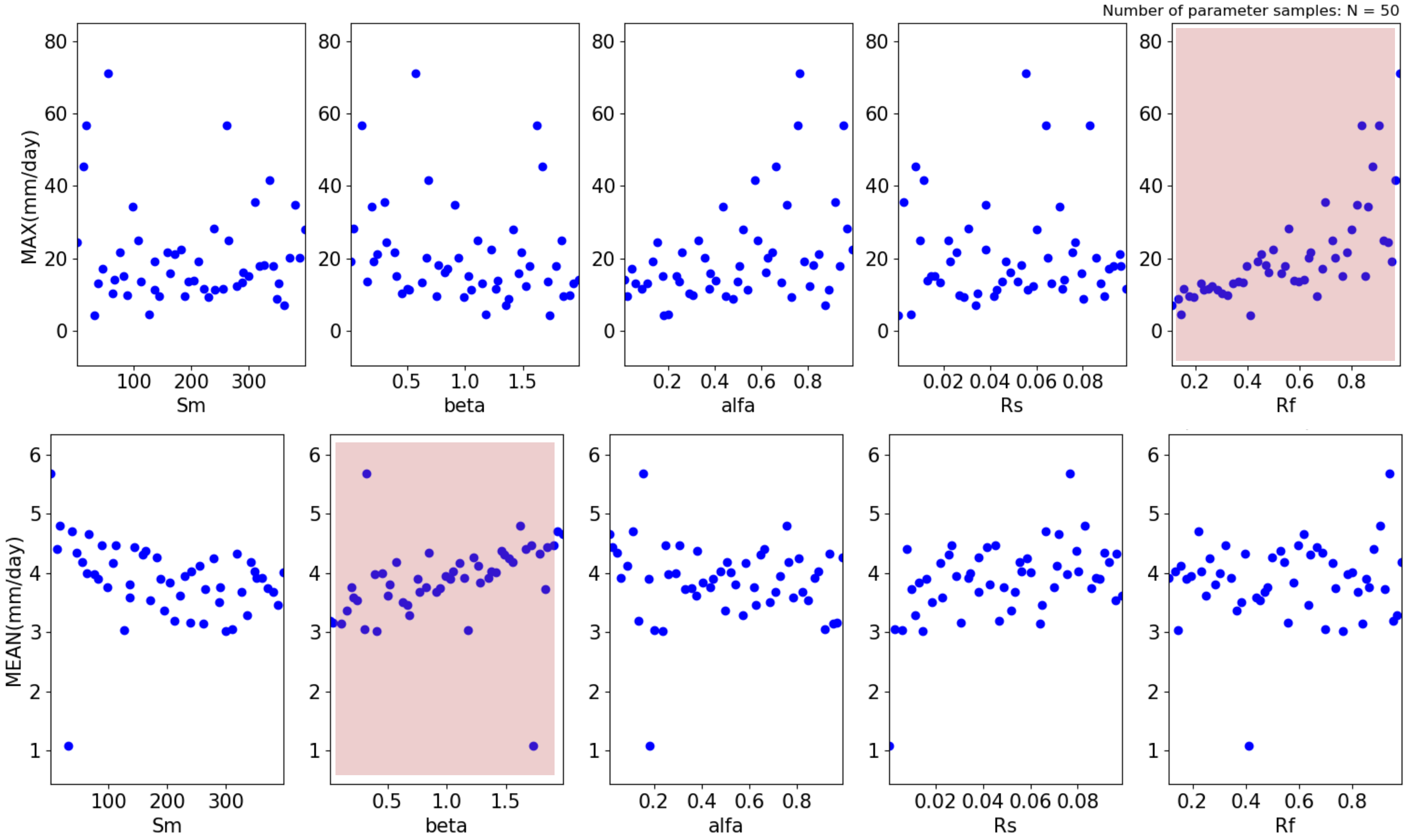
metric MAX



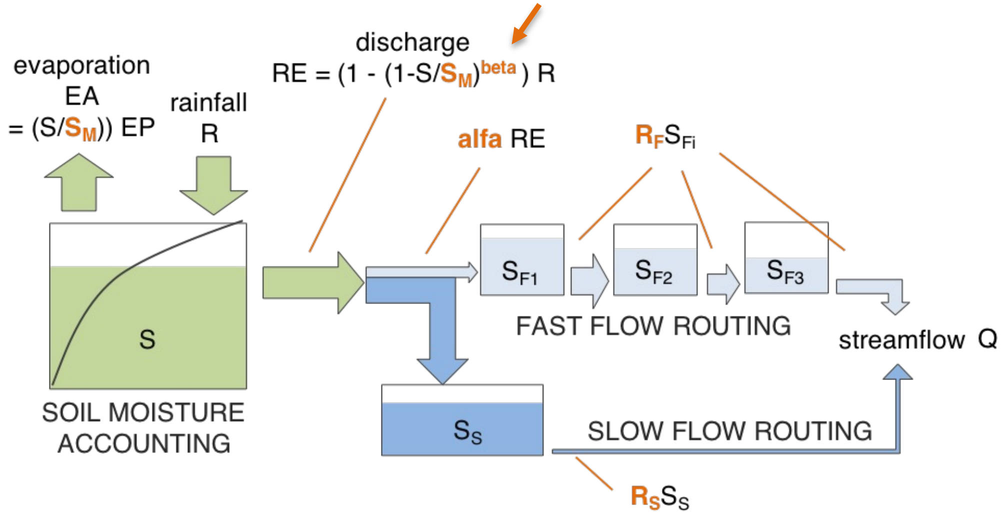
Analyzing input-output dataset

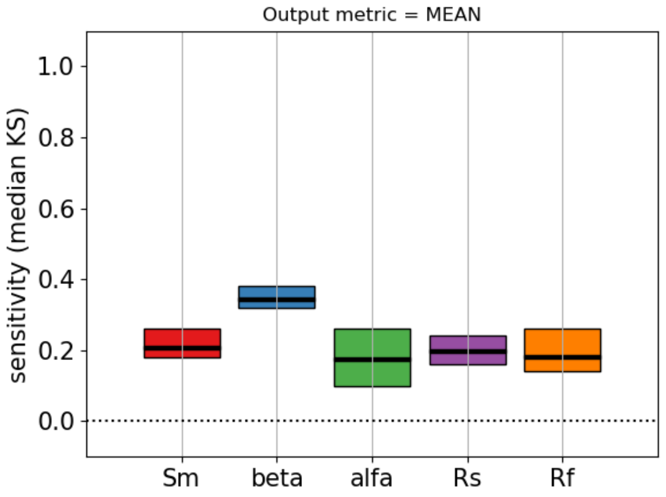
Flood defences
max flow

Water supply systems
mean flow

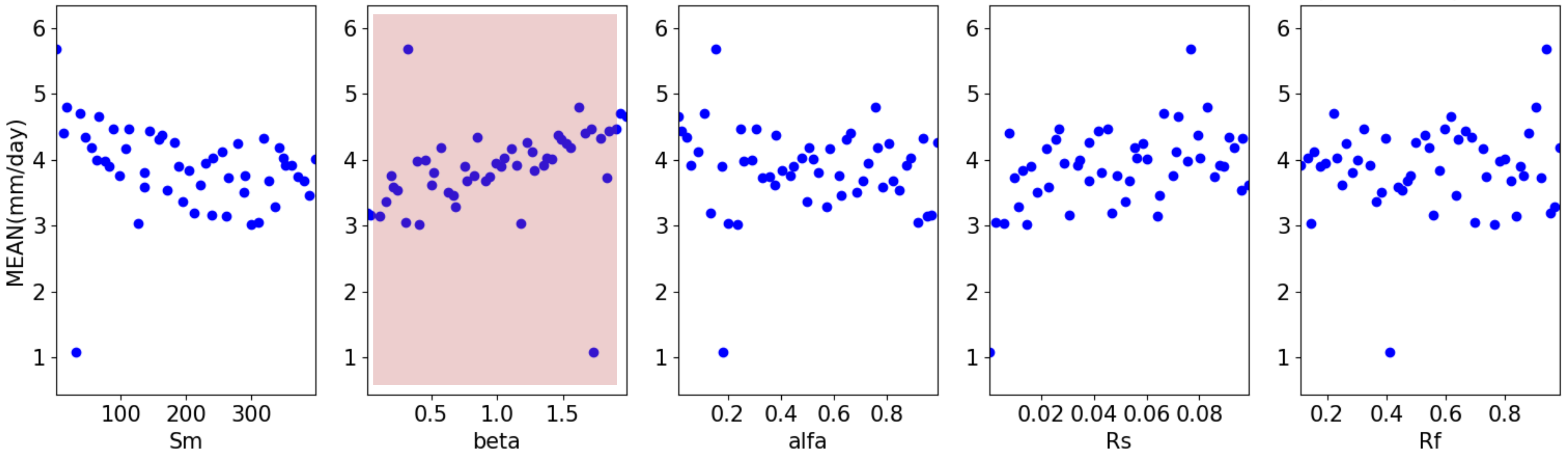


Analyzing input-output dataset



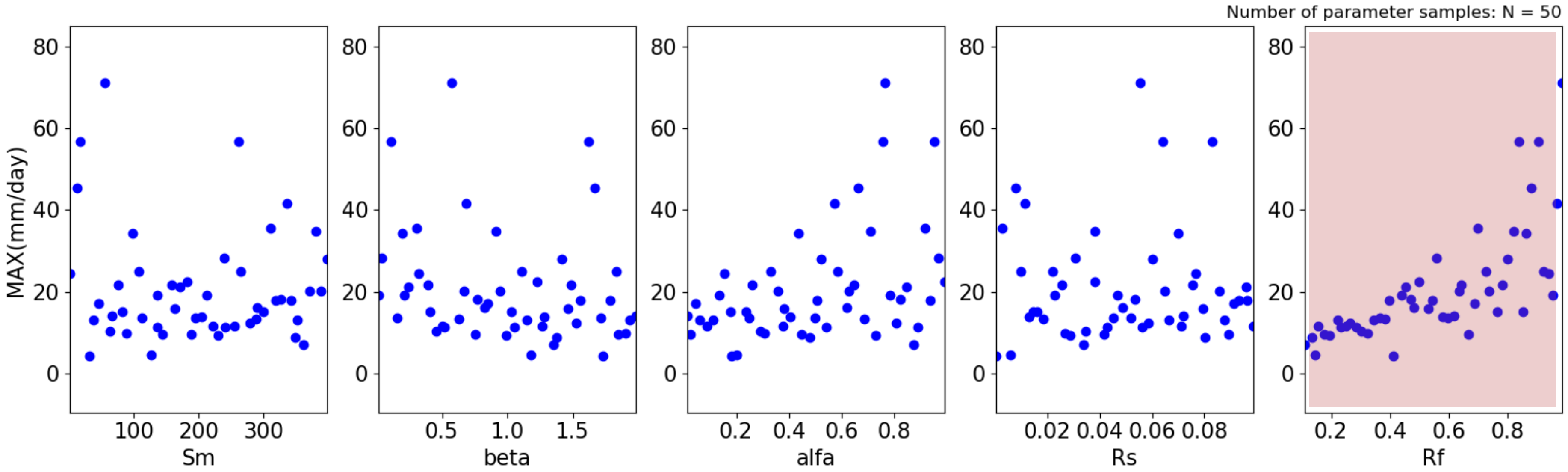


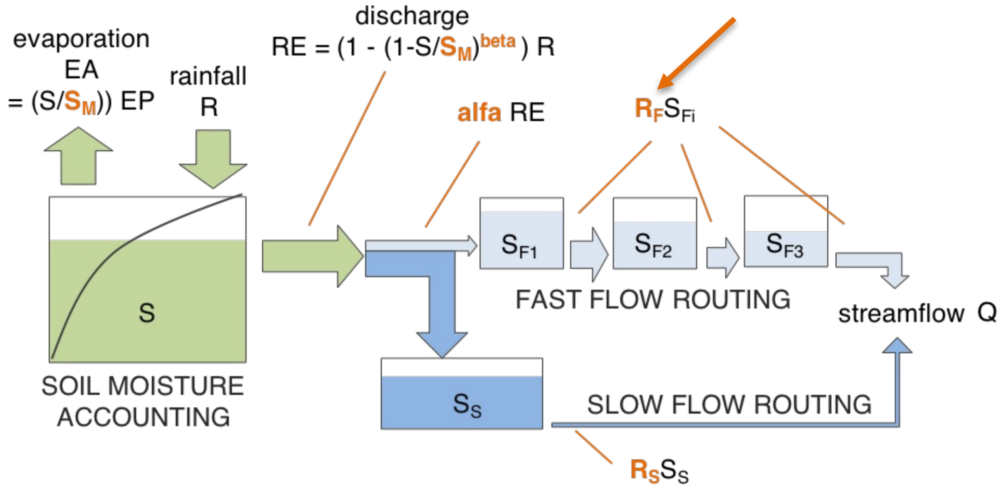
Water supply systems
mean flow

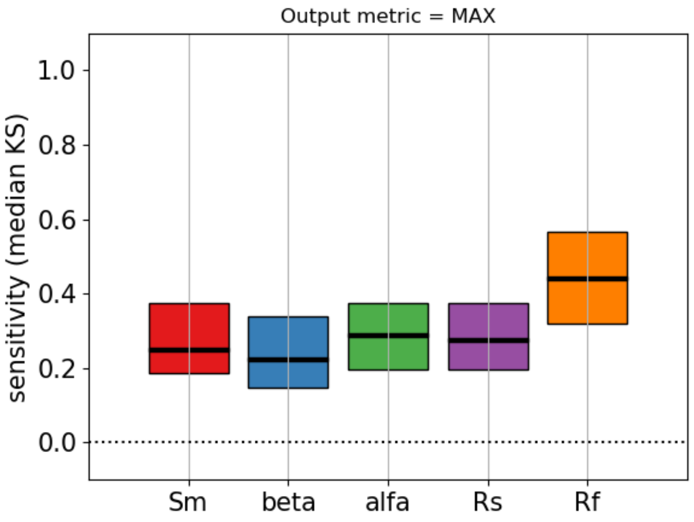


Analyzing input-output dataset

Flood defences
max flow







Uncertainty Quantification and Sensitivity Analysis for Resilient Infrastructure Systems (USARIS)

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Ultimately USARIS will contribute to enable and promote best practices for responsible modelling in the DAFNI users community

Get in touch if you want to:

- learn more about UA&SA
- brainstorm ideas on how UA&SA can help in your sector
- discuss training opportunities
- provide pilot applications

Pianosi
UQ&SA
water systems

Salwey
UQ&SA
water systems

Bloomfield
energy systems

Coxon
water systems

How

to do UA/SA?

Characterizing uncertainty in input factors

What is the appropriate distribution / range for the uncertain inputs?

characterize uncertainty in the inputs	sample N combinations of inputs	execute the model against each inputs' combination	calculate output(s) ranges or distribution	calculate sensitivity indices

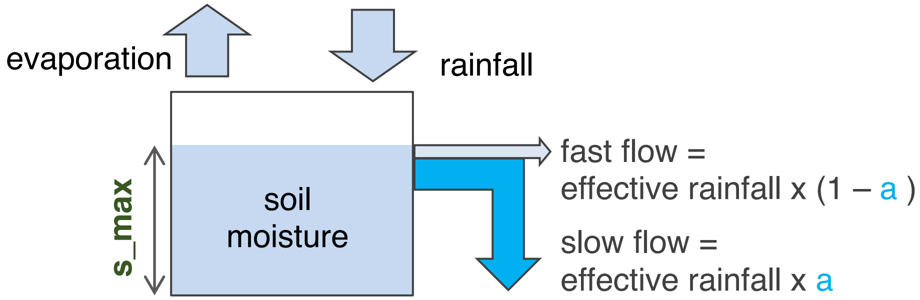
Characterizing uncertainty in input factors

Depending on the type of input (scalar parameter, time series, discrete modelling choice, etc.) and on our level of uncertainty about it, we can use:

- a list of possible values
    - a uniform distribution within an uncertainty range
        - a probability distribution

… and define them using literature sources, historical observations, experts' judgment, etc.

Sometimes the range (distribution) is univocally defined by the physical meaning of the input, but most often different definitions are possible

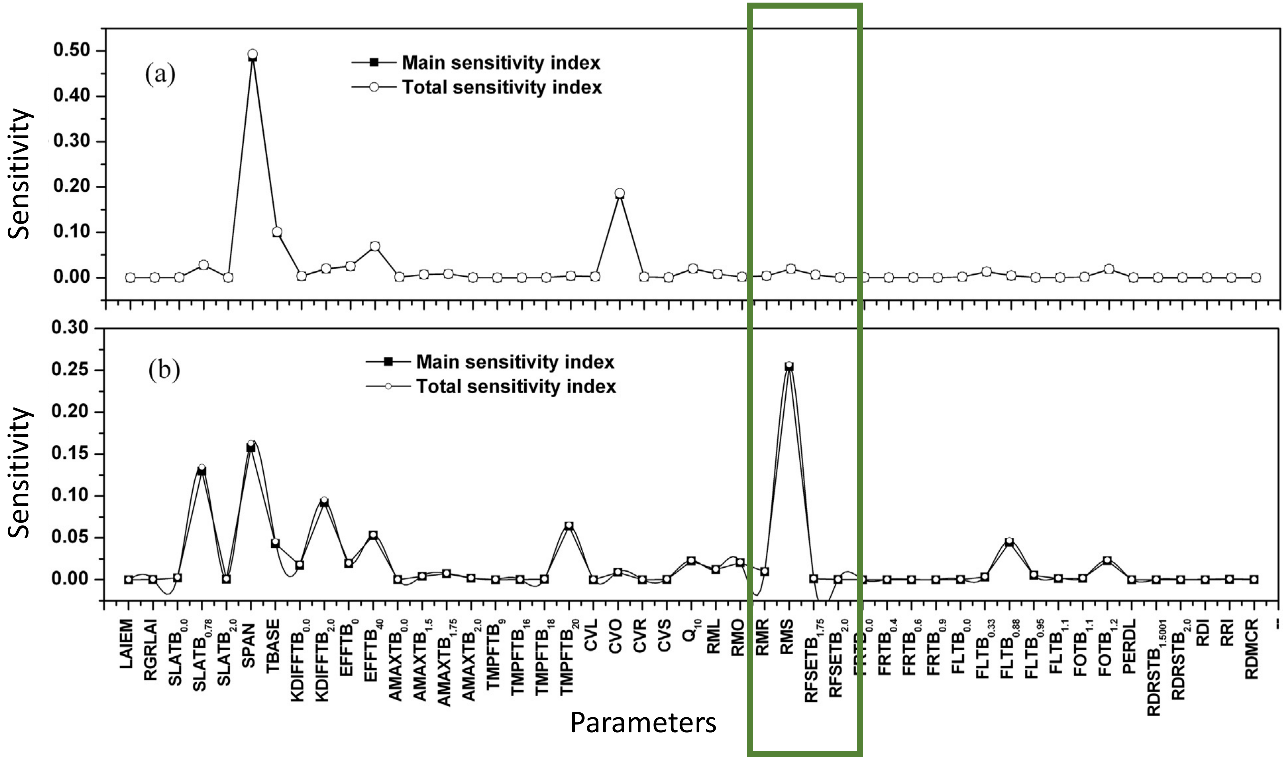


The maximum soil capacity (s_max) varies between 0 and an upper bound that may be difficult to define

The repartition coefficient (a) varies between 0 and 1 by definition

When different definitions of the inputs ranges (distributions) are possible, the choice can significantly condition UA/SA results

Example from SA of a crop growth model (Wang et al EMS 2013)



Sensitivity of simulated crop yield based on 'literature' parameter ranges

Sensitivity of simulated crop yield based on 'expanded' parameter ranges

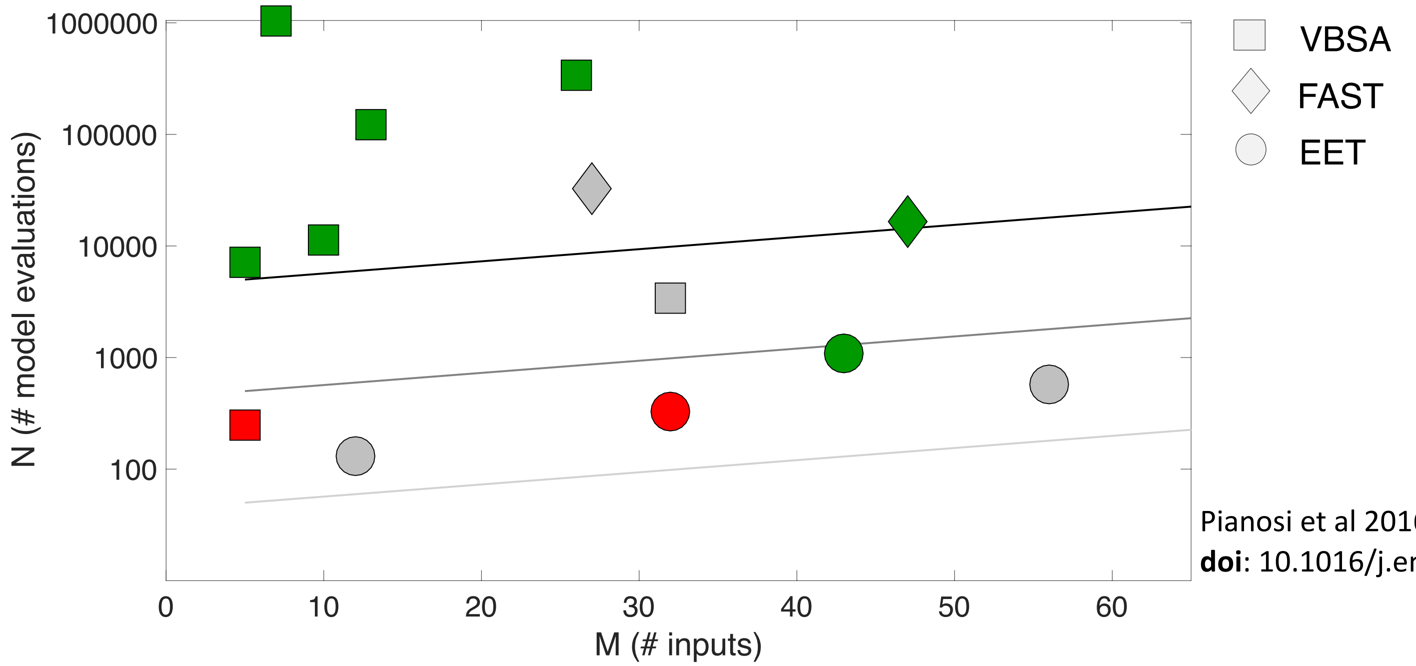
Choosing the sampling strategy and size

Which sampling technique to use (e.g. random sampling, Latin Hypercube, quasi-random sequences)?

How many samples are needed?

characterize uncertainty in the inputs	sample N combinations of inputs	execute the model against each inputs' combination	calculate output(s) ranges or distribution	calculate sensitivity indices

In general, the required sample size (N) increases with the number of uncertain inputs (M). However, the proportionality rate varies significantly from one method to another, and from one application of the same method to another

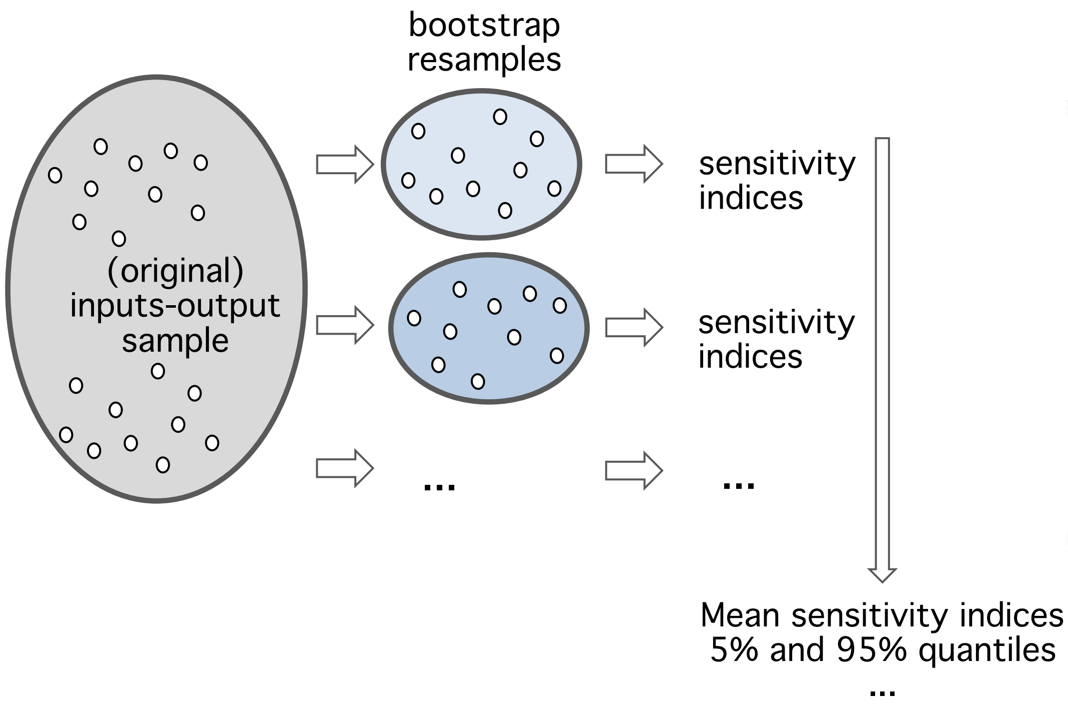


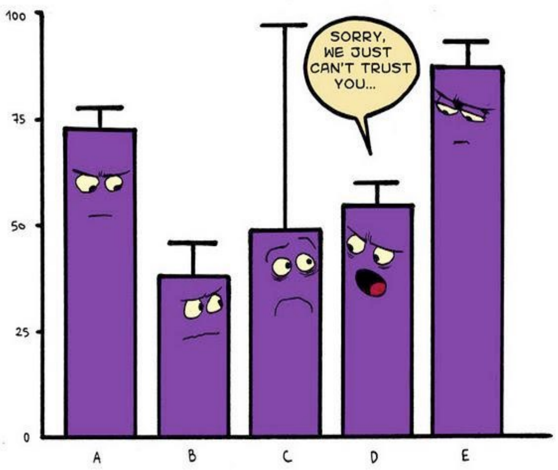
Pianosi et al 2016

doi: 10.1016/j.envsoft.2016.02.008

Sensitivity indices are calculated from a sample, so if the sample size is small, their values may be poorly approximated

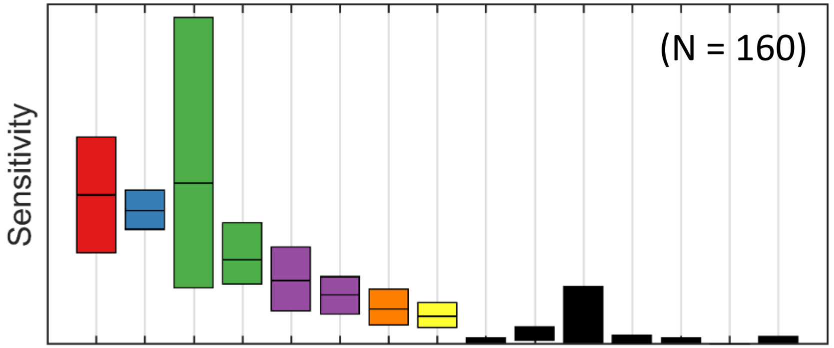
In order to assess the robustness of our sensitivity estimates to the chosen sample, *without re-running the model*, we can use bootstrapping



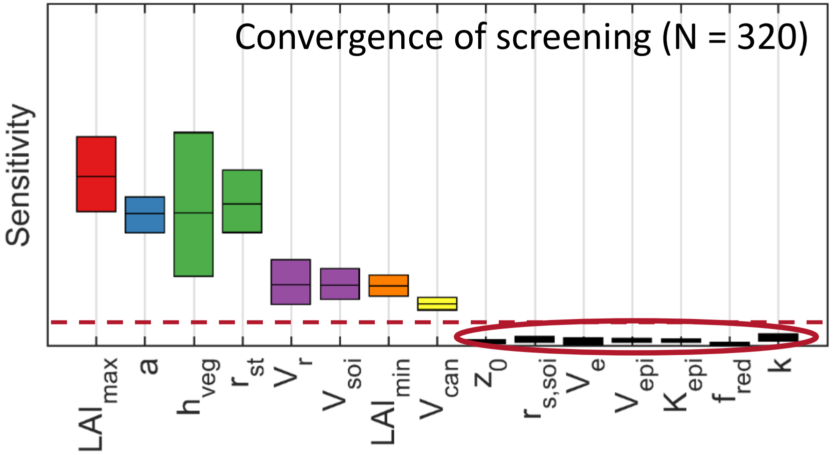


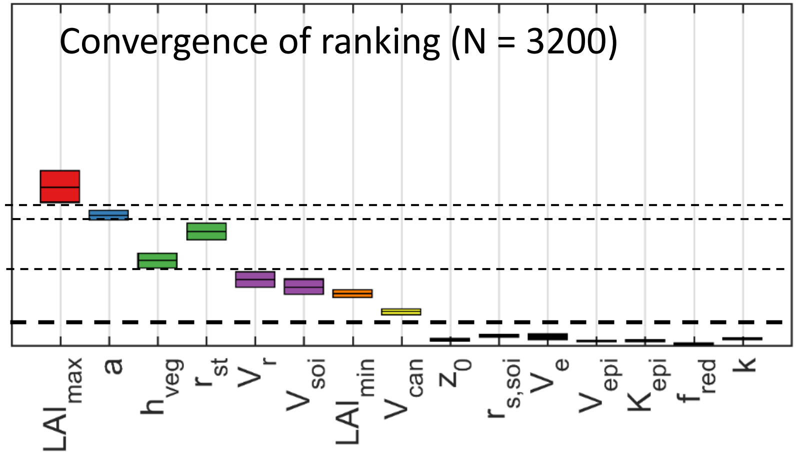
SOURCE: Reading Local Group of the Royal Statistical Society

If the confidence intervals of our sensitivity indices are not “small enough” we must increase the sample size (what is “small enough” depends on the goal of our GSA)



black line: mean sensitivity index
bar: 90% confidence interval





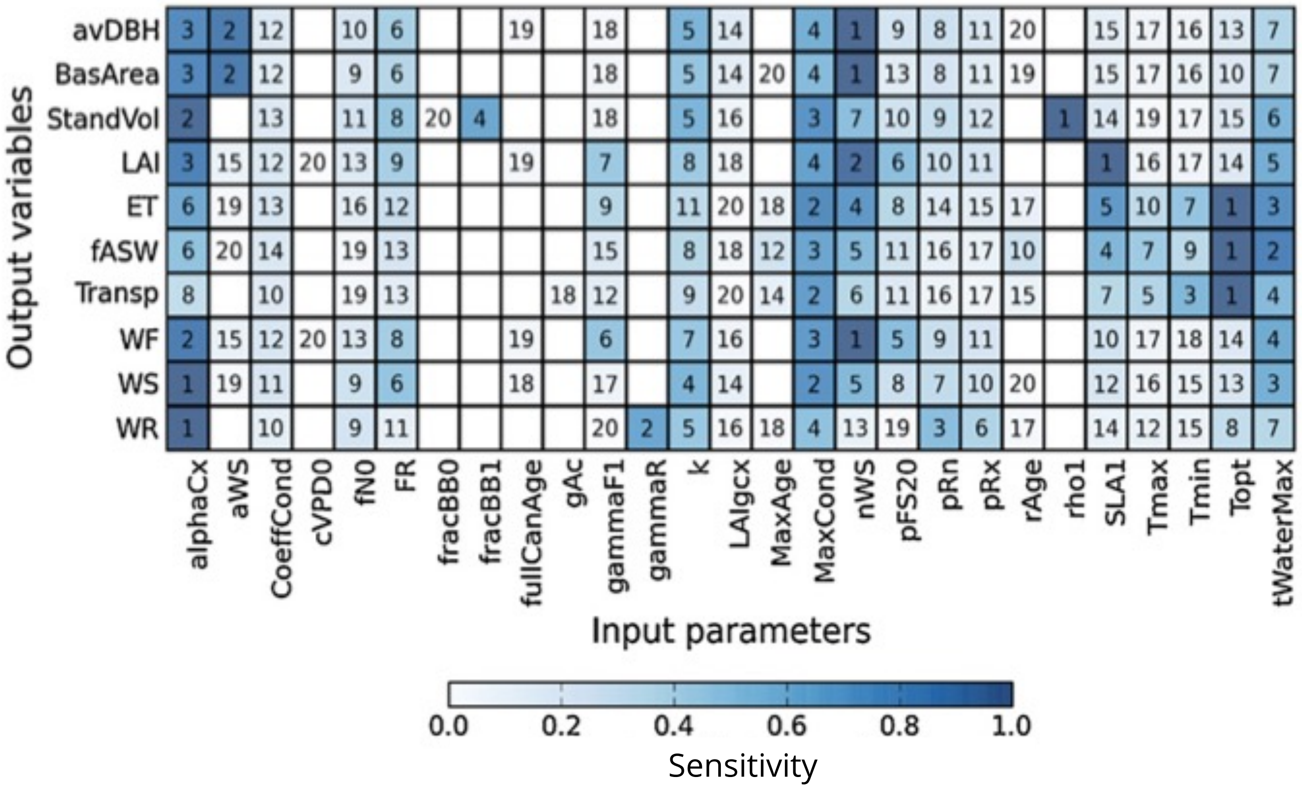
Defining scalar output metric(s)

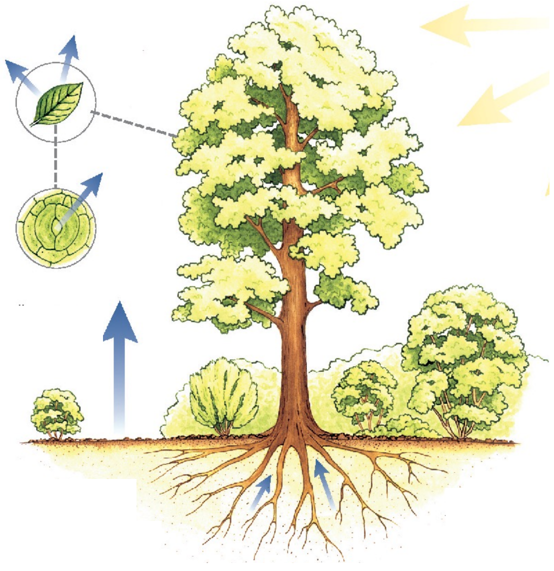
Which output metrics should we look at?

characterize uncertainty in the inputs	sample N combinations of inputs	execute the model against each inputs' combination	calculate output(s) ranges or distribution	calculate sensitivity indices

Each output metric is typically sensitive to a small subset of inputs, but which are those inputs will differ from one metric to another

Application to a forest growth model (Song et al EcM 2012)





Choosing a method for calculating sensitivity indices

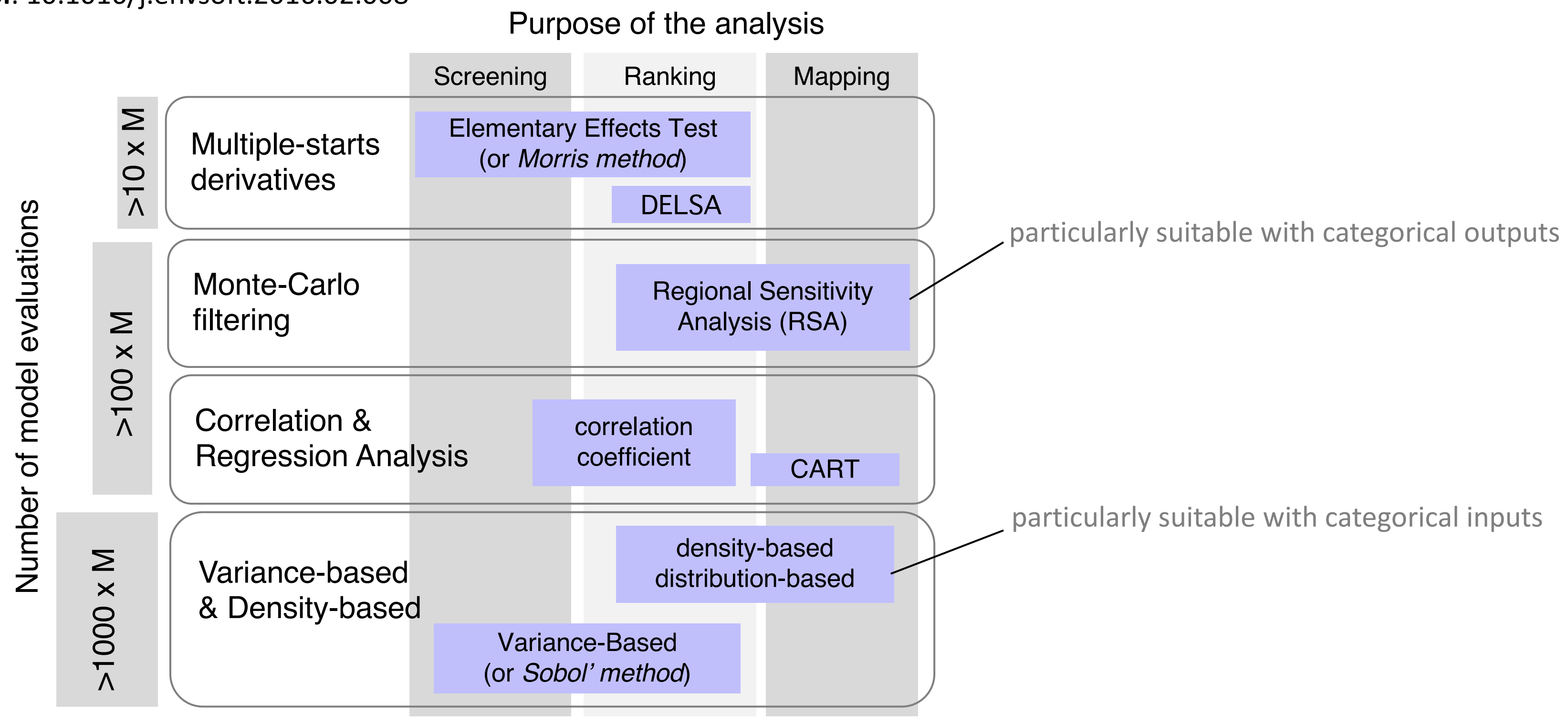
Which global sensitivity analysis method to use (e.g. variance-based, elementary effects test, regional sensitivity analysis, etc.)?

characterize uncertainty in the inputs	sample N combinations of inputs	execute the model against each inputs' combination	calculate output(s) ranges or distribution	calculate sensitivity indices

Different methods defines "sensitivity" in different ways and are more or less suitable for specific purposes or problems

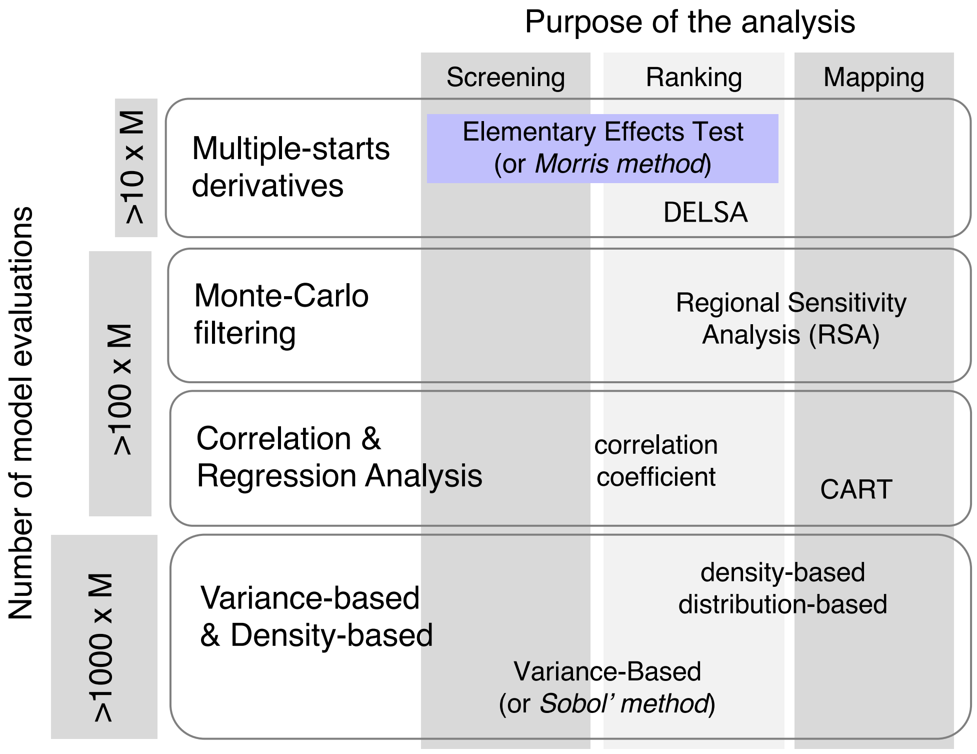
Pianosi et al 2016

doi: 10.1016/j.envsoft.2016.02.008



M = number of input factors

Elementary Effects Test
(Morris, 1991)

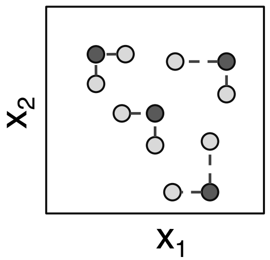


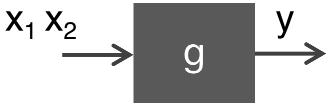
Sensitivity is proportional to...

the mean finite differences of the output across the input space

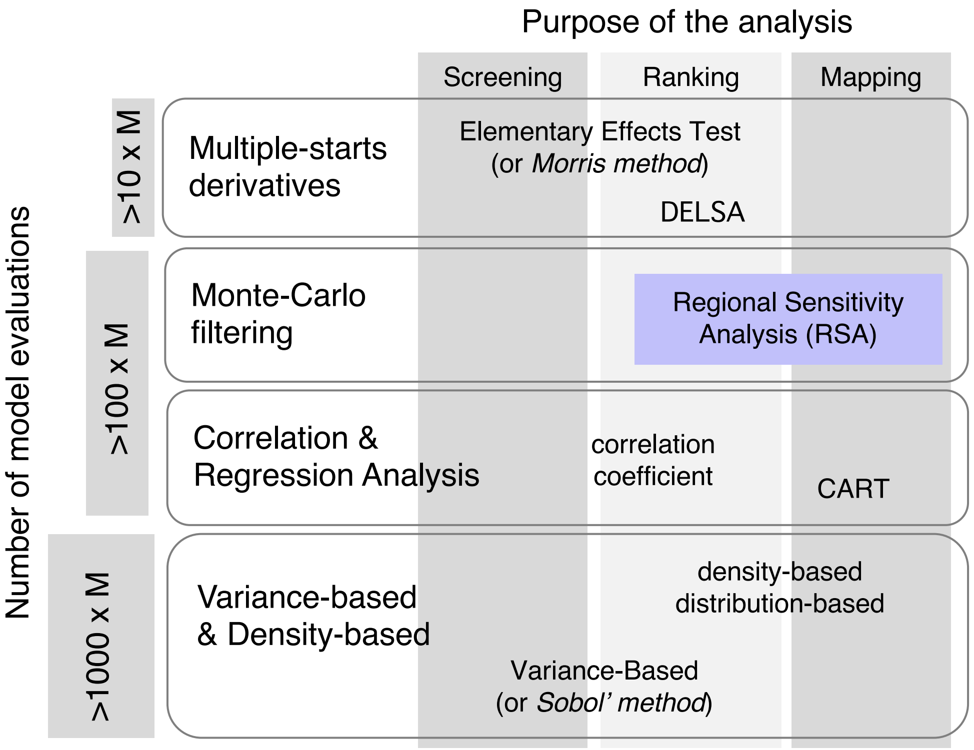
$$S_i = \frac{1}{r}\sum_{j=1}^{r} EE^j$$

$$EE^j = \frac{g(\bar{x}_1^j, ..., \bar{x}_i^j + \Delta_i^j, ..., \bar{x}_M^j) - g(\bar{x}_1^j, ..., \bar{x}_i^j, ..., \bar{x}_M^j)}{\Delta_i^j} c_i$$



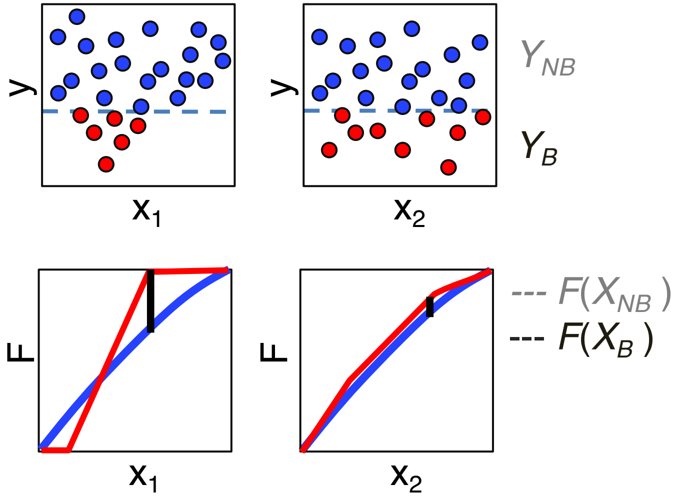


Regional Sensitivity Analysis (Hornberger & Spear, 1980)

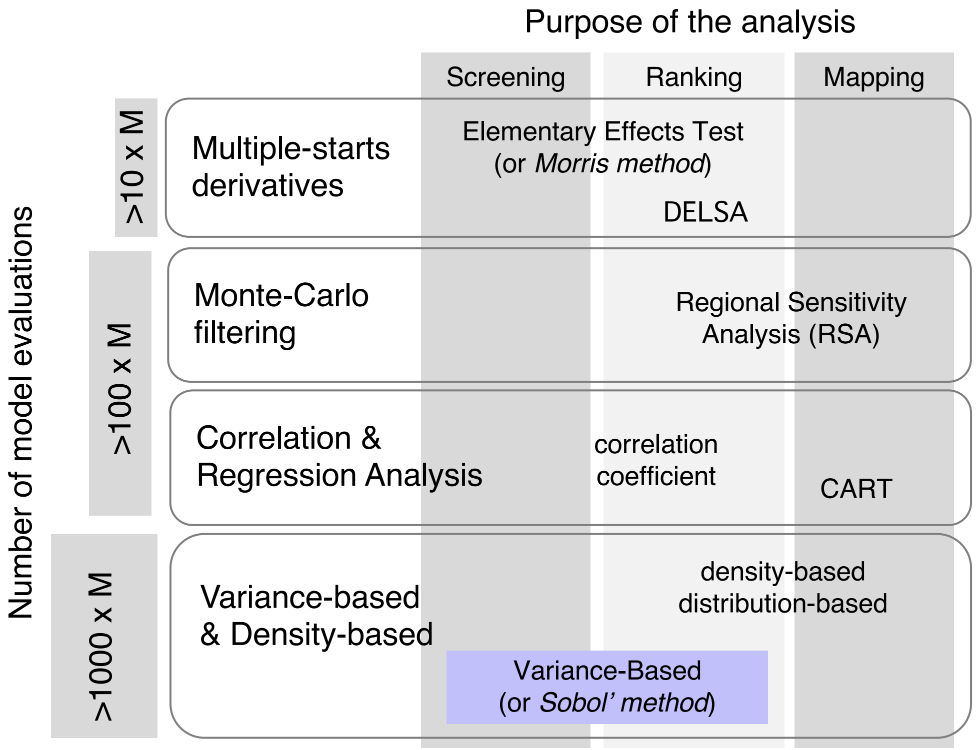


Sensitivity is proportional to...
the variation induced in the distribution of an input by conditioning the output

$$S_i = \max_q \left| F_{x_i}(x|x \in X_B) - F_{x_i}(x|x \in X_{NB}) \right|$$



Variance-based Sensitivity Analysis (Homma & Saltelli, 1996)



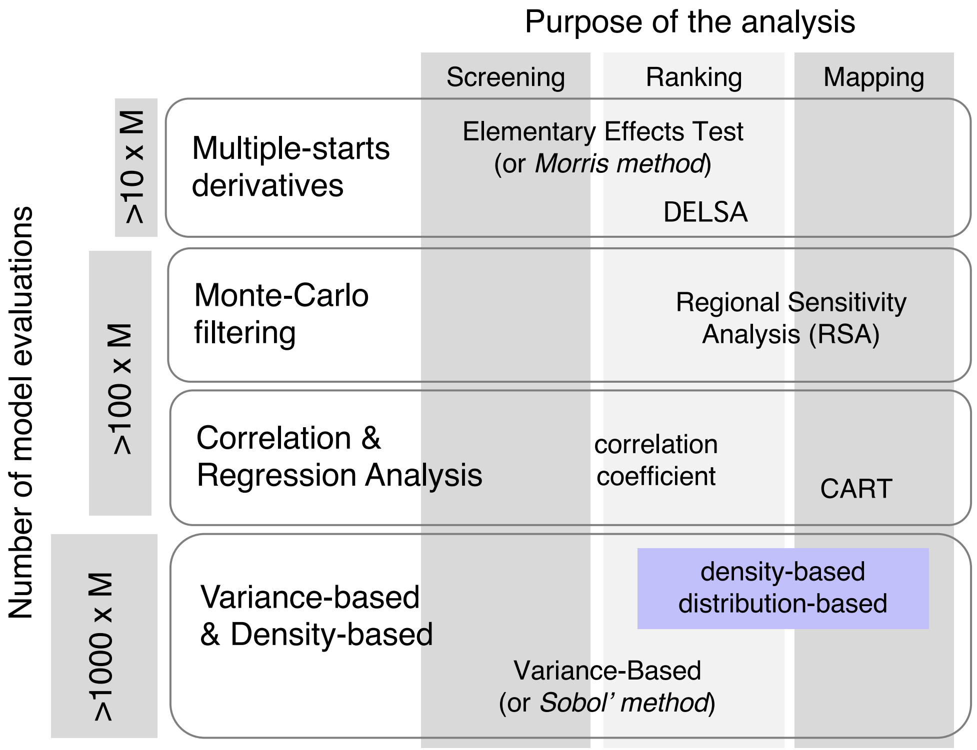
Sensitivity is proportional to...
variation induced in the variance of the output by conditioning an input

$$S_i = \frac{V_i}{V} = \frac{V_{x_i}[E(y|x_i)]}{V(y)} = \frac{V(y) - E_{x_i}[V(y|x_i)]}{V(y)}$$

$$S_i^T = 1 - \frac{V_{x_{\sim i}}}{V} = 1 - \frac{V_{x_{\sim i}}[E(y|x_{\sim i})]}{V(y)}$$

$$V = \sum_i V_i + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots + V_{12\ldots M}$$

Distribution-based Sensitivity Analysis (PAWN)
(Pianosi & Wagener 2015)



Sensitivity is proportional to...

variation induced in the distribution of the output by conditioning an input

$$S_i = \underset{k}{\mathrm{stat}} \ \max_{q} \left| F_y(q) - F_{y|x_i}(q|x_i \in I_k) \right|$$